MATH 346/Spring 2005 **TEST II**

(1) The binary search algorithm is
function(input: integer x; list of sorted integers $a_1 \leq \cdots \leq a_n$)
i=1
j=n
while($i < j$)
{ $m = \lfloor \frac{i+j}{2} \rfloor$
if $x > a_m$ then $i = m + 1$ else $j = m$}
if $x = a_i$ then $location = i$ else $location = 0$

Suppose that the list entered is $1, 3, 3, 5, 8, 13, 14, 17$ and the value of x is 13. Show the steps that the pseudo-code performs to locate x.

(2) Prove that the function $f(x) = 2x^3 + 4x^2 + x + 5$ is $O(x^3)$. Show carefully the witnesses C and k of the proof.

(3) Prove that $1^2 + 2^2 + 3^2 + \cdots + n^2$ is $O(n^3)$. Show carefully the witnesses C and k of the proof.

(4) (a) Find the remainder of the division of 11223344556677889 by 3
(b) Find $\gcd(410, 103)$ by applying the Euclidean algorithm.

(5) Find the sums:
(a) $\sum_{i=1}^{25} (-\frac{1}{2})^i$
(b) $\sum_{i=1}^{200} (3i + 2)$

(6) (a) represent the decimal number 1021 in base 2
(b) represent the number $(A2FC)_{16}$ in base 10
(c) represent the number $(A2FC)_{16}$ in base 2

(7) Prove by mathematical induction that 6 divides $n^3 - n$ whenever n is a nonnegative integer.

(8) Suppose that the only currency were 4-dollar bills and 9-dollar bills. Show that every amount greater than k dollars could be made from a combinations of these bills, you need to specify the value of k.

(9) Show that the sequence of primes is infinite.